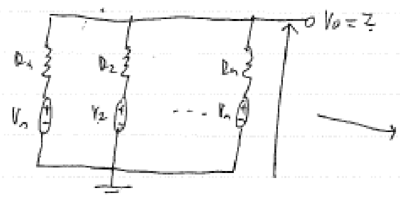
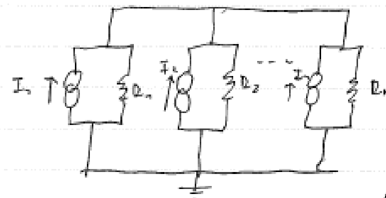
Exam-solution-finalApril-2014

Problem-1

Problem 1a:

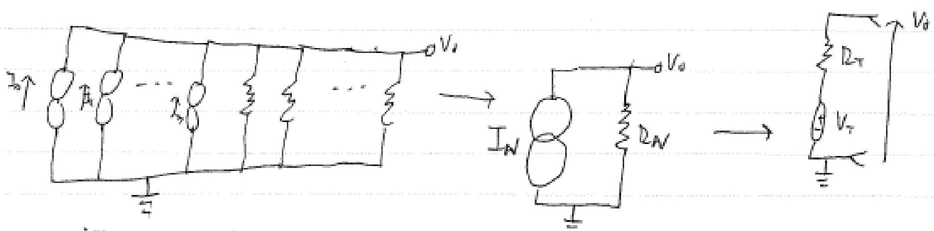


We use Norton equivalents:



$$I_i = \frac{V_i}{R_i}$$

resistors are parallel, current sources also

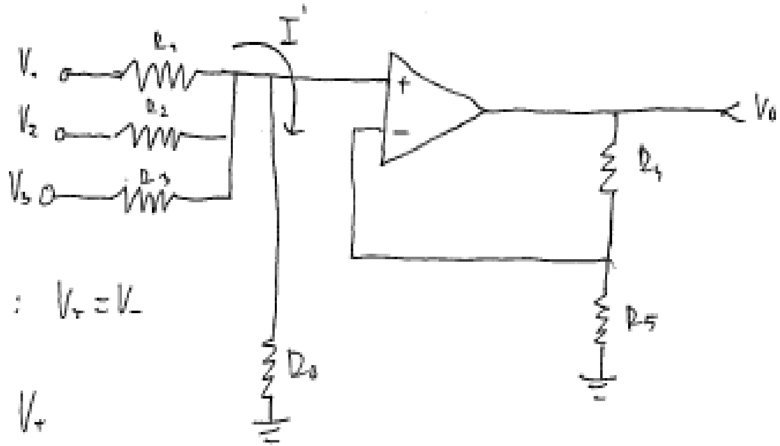


$$I_N = \sum_{i}^{i=n} I_i = \sum \frac{V_i}{R_i}$$

$$R_N = \frac{1}{\sum \frac{1}{R_i}} = R_T$$

$$V_0 \overset{=V_T}{=} I_N R_N = \frac{\sum_{i=1}^{n} \frac{V_i}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

Problem 2a) :



negative feedback : $V_+ = V_-$

$$V_- = V_0 \frac{R_5}{R_5 + R_4} = V_+$$

$$I' = \frac{V_1 - V_+}{R_1} + \frac{V_2 - V_+}{R_2} + \frac{V_3 - V_+}{R_3} = \frac{V_+}{R_0} = \frac{V_1}{R_1} - \frac{V_+}{R_1} + \frac{V_2}{R_2} - \frac{V_+}{R_2} + \frac{V_3}{R_3} - \frac{V_+}{R_3} \rightarrow V_+ \left(\frac{1}{R_0} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

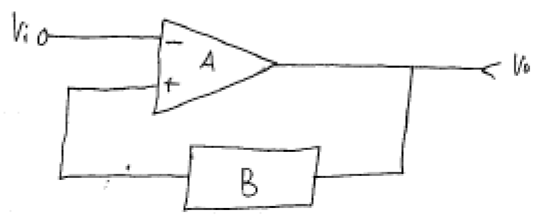
$$V_0 \frac{R_5}{R_5 + R_4} \cdot \left(\sum \frac{1}{R_i} + \frac{1}{R_0} \right) = \sum \frac{V_i}{R_i}$$

$$\boxed{V_0 = \left(1 + \frac{R_4}{R_5}\right) \frac{\sum \frac{V_i}{R_i}}{\frac{1}{R_0} + \sum \frac{1}{R_i}}}$$

or explicitly $V_0 = \left(1 + \frac{R_4}{R_5}\right) \dfrac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_0}}$

↳ probably very big

Problem 1b) :



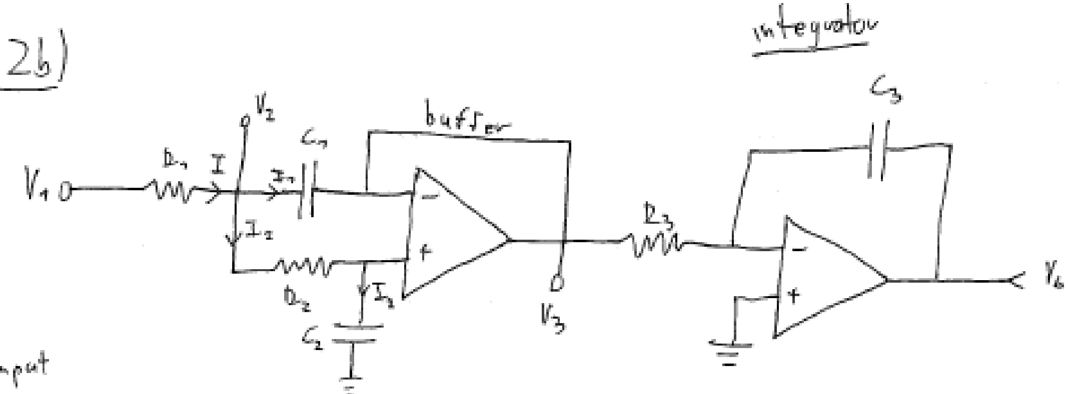
→ positive feedback : $V_o = A(V_+ - V_-)$; $V_+ = BV_o$, $V_- = V_i$

$V_o = ABV_o - AV_i$ → $V_o(1-AB) = -AV_i$

→ $G = \frac{V_o}{V_i} = \frac{-A}{1-AB}$ → oscillations when $AB = 1$

→ $Re(AB) = 1$, $Im(AB) = 0$ } $\varphi = 0°$, $AB = 0\,dB$

Problem 2b)



$J_i = R_i C_i$

$V_2(V_1, V_3) = ?$ (input)

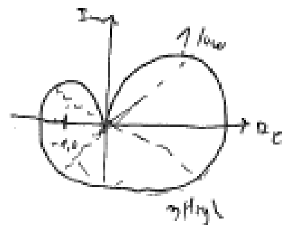
$V_3 = V_1 \rightarrow$ $\quad I = I_1 + I_2 \rightarrow \frac{V_1 - V_2}{R_1} = \frac{V_2 - V_3}{Z_{C_1}} + \frac{V_2 - V_3}{R_2} = (V_2 - V_3)\left(i\omega C_1 + \frac{1}{R_2}\right)$

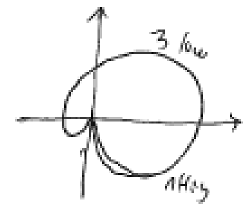
$V_1 - V_2 = (V_2 - V_3)\left(i\omega R_1 C_1 + \frac{R_1}{R_2}\right)$

$V_2\left(1 + \frac{R_1}{R_2} + i\omega J_1\right) = V_1 + V_3\left(\frac{R_1}{R_2} + i\omega J_1\right)$

$$V_2 = \frac{V_1 + V_3\left(\frac{R_1}{R_2} + i\omega J_1\right)}{1 + \frac{R_1}{R_2} + i\omega J_1}$$

Problem 3a)





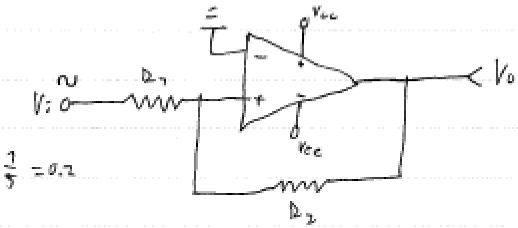
System (a) has:

- 1 low cut-off
- 3 high cut-offs
- very unstable (possible osscillations)
  ↳ at high frequencies

System (b) has:

- 3 low
- 1 high
- unstable at low frequencies, but no oscillations
  (↗ positive feedback)

Problem 3b) Schmitt trigger



$V_{cc} = 15V$

$V_{ee} = -15V$

$R_1 = 3k$

$R_2 = 15k$

$\left.\right\} \frac{R_1}{R_2} = \frac{1}{5} = 0.2$

$V_i = 5V \cdot \sin(\omega t)$

$$\frac{V_i - V_+}{R_1} = \frac{V_+ - V_0}{R_2} \rightarrow \frac{V_+}{R_2} + \frac{V_+}{R_1} = \frac{V_i}{R_1} + \frac{V_0}{R_2}$$

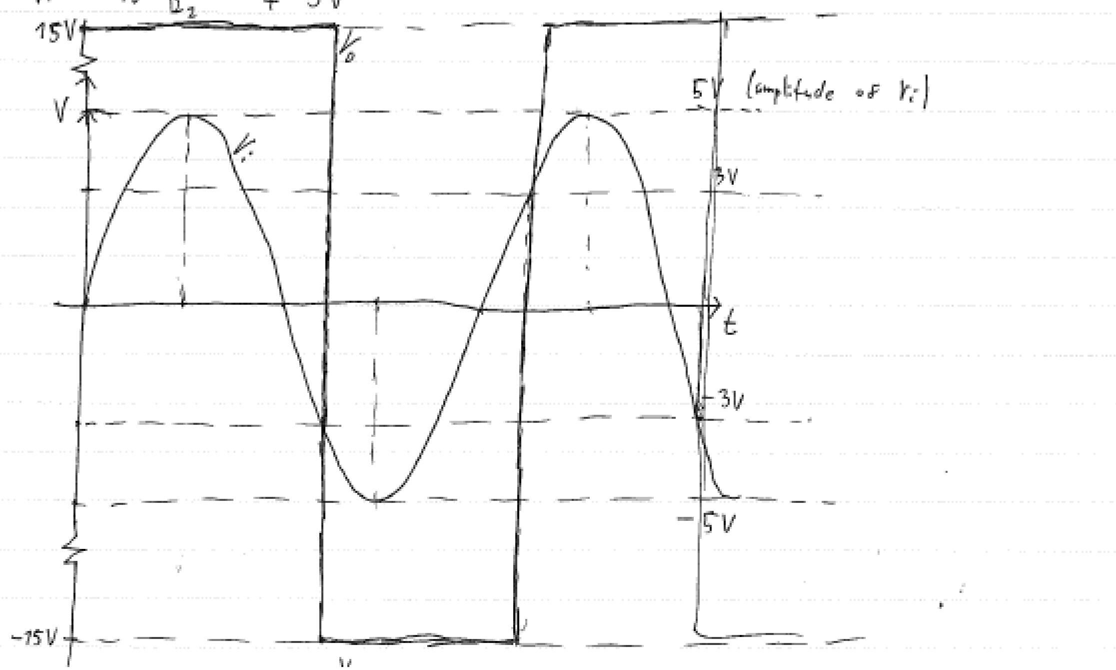
$$V_+ \frac{R_1 + R_2}{R_1 R_2} = \frac{V_i R_2 + V_0 R_1}{R_1 R_2}$$

$$\boxed{V_+ = V_i \frac{R_2}{R_1 + R_2} + V_0 \frac{R_1}{R_1 + R_2}}$$

$V_0 = \pm 15V \rightarrow$ it switches when $V_+ = 0$

$$V_+ = V_i \cdot \frac{15}{18} + V_0 \frac{3}{18} = V_i \frac{5}{6} + V_0 \frac{1}{6} = \frac{5}{6} V_i \pm 2.5V$$

$$\rightarrow V_i = -V_0 \frac{R_1}{R_2} = \mp 3V$$



Problem 4a)

$1 \rightarrow 4$ counter

	Q_3	Q_2	Q_1	Q_3	Q_2	Q_1	J_3	K_3	J_2	K_2	J_1	K_1
1	0	0	1	0	1	0	0	x	1	x	x	1
2	0	1	0	0	1	1	0	x	x	0	1	x
3	0	1	1	1	0	0	1	x	x	1	x	1
4	1	0	0	0	0	1	x	1	0	x	1	x

$J_1 = K_1 = 1$

J_3:

Q_3 \ Q_2Q_1	00	01	11	10
0	X	0	1	0
1	X	X	X	X

K_3:

	00	01	11	10
0	X	X	X	X
1	1	X	X	X

J_2:

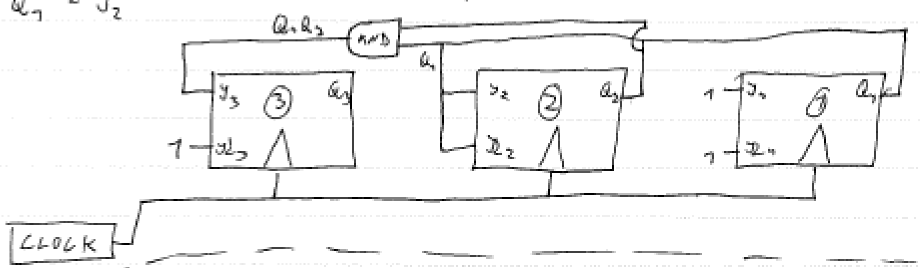
	00	01	11	10
0	X	1	X	X
1	0	X	X	X

K_2:

	00	01	11	10
0	X	X	1	0
1	X	X	X	X

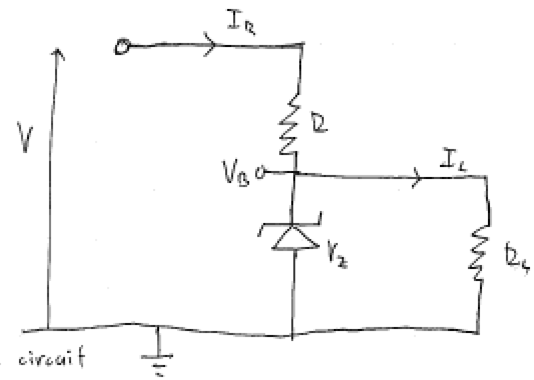
$J_3 = Q_2 Q_1$, $K_3 = 1$ / could also put $J_2 = \overline{Q_3}$

$J_2 = Q_1$, $K_2 = Q_1 = J_2$



Problem (4b)

$V > V_Z$



when Zener doesn't conduct (→ open circuit): $I_R = I_L$ $\qquad \frac{V - V_B}{R} = \frac{V_B - 0}{R_L}$

$V_B = V \frac{R_L}{R + R_L} < V_Z$

$\Rightarrow V < V_Z \left(1 + \frac{R}{R_L}\right)$ / $\boxed{V_Z < V < V_Z \left(1 + \frac{R}{R_L}\right)}$

$\frac{V}{V_Z} < 1 + \frac{R}{R_L} \qquad \frac{R}{R_L} > \frac{V}{V_Z} - 1$

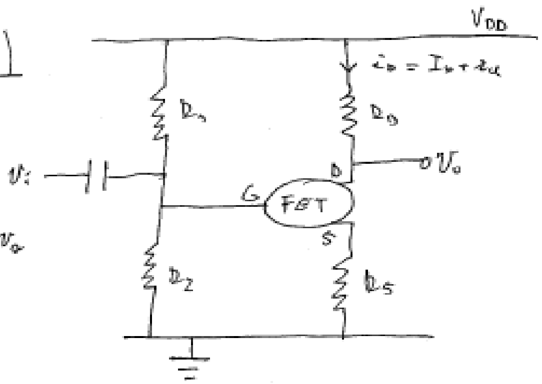
$\boxed{R > R_L \left(\frac{V}{V_Z} - 1\right)}$ → always positive because $V > V_Z$

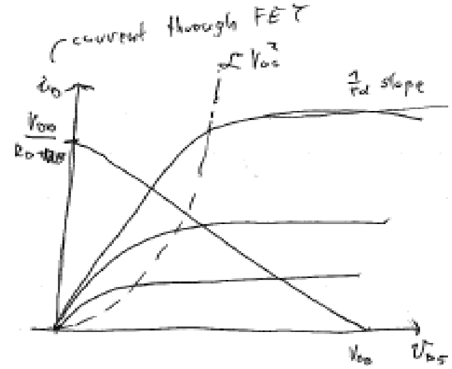
Problem 5a)

$v_G = V_G + v_g$

$v_D = V_D + v_d$

$v_S = V_S + v_s$





~~if $V_{GS} \leq V_T$ ... FET ... $i_D = 0$~~

$V_G = V_{DD} \frac{R_2}{R_1+R_2}$ (for biasing) $\rightarrow v_g = v_i$ (small signal)

when $V_{GS} > V_T$, transistor conducts through R_D and R_S $\quad i_D = \frac{V_{DD}}{R_D+R_S}$

$I_D = \frac{V_{DD}}{R_D+R_S} \rightarrow V_S = 0 + I_D R_S = V_{DD}\frac{R_S}{R_D+R_S}$

$i_D = i_D(v_{GS}, v_{DS}) = i_D(V_{GS}, V_{DS}) + \frac{\partial i_D}{\partial v_{GS}} v_{gs} + \frac{\partial i_D}{\partial v_{DS}} v_{ds} + \ldots$

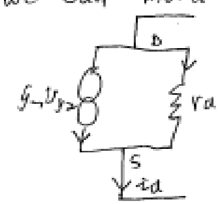
follows from Taylor expansion

$$\boxed{i_D = I_D + g_m v_{gs} + \frac{1}{r_d} v_{ds}}$$

$v_{gs} = v_g - v_s = v_i - i_d R_S$

$v_{ds} = v_d - v_s = v_o - i_d R_S$

we can model this by equivalent



$i_d =$

comes from $i_D(v_{GS}) = \frac{k}{2}(v_{GS} - V_T)^2$

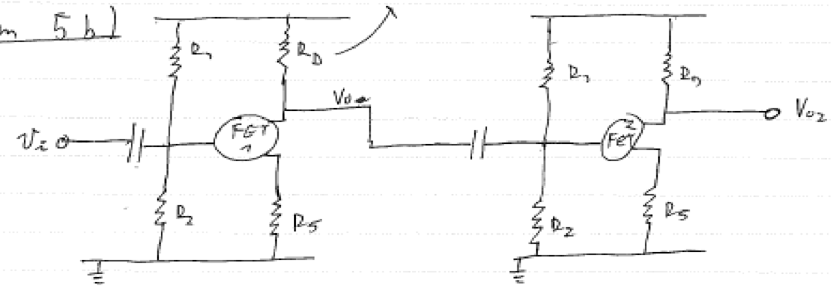
$\frac{\partial i_D}{\partial v_{GS}} = k(V_{GS} - V_T)$ is a constant g_m

comes from the slope of the saturation region in $i_D(v_{DS})$ characteristic

v_i affects i_D by $g_m v_{gs}$ which affects v_{ds} which affects back i_D → that's the motivation for both terms $\frac{\partial i_D}{\partial v_{GS}}$ and $\frac{\partial i_D}{\partial v_{DS}}$

probably meant R_D

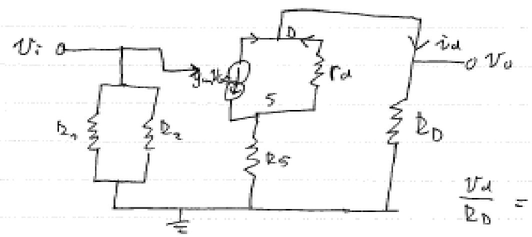
Problem 5 b)



$G_1 = G_2$

$$\frac{v_{o2}}{v_i} = \frac{v_{o2}}{v_{o1}} \cdot \frac{v_{o1}}{v_i}$$ / two transistor circuits are identical, so the gains are the same $G = G_1 \cdot G_2 = G'^2$

$G' = \frac{v_o}{v_i}$ → we use small signal analysis:



$v_g = v_i$, $v_o = v_d$

$$i_d = \frac{v_d}{R_D} = -g_m(v_i - v_s) + \frac{v_s - v_d}{r_d}$$

$$\frac{v_d}{R_D} = \frac{0 - v_s}{R_S} \rightarrow \boxed{v_s = -v_d \frac{R_S}{R_D}}$$

$$v_d = -g_m R_D v_i + g_m R_D v_s + v_s \frac{R_D}{r_d} - v_d \frac{R_D}{r_d}$$

$$v_d\left(1 + \frac{R_D}{r_d}\right) = -g_m R_D v_i - v_d R_S\left(g_m + \frac{1}{r_d}\right)$$

$$v_d\left(1 + \frac{R_D}{r_d} + g_m R_S + \frac{R_S}{r_d}\right) = -g_m R_D v_i \quad (v_d = v_o)$$

$$\boxed{\frac{v_o}{v_i} = \frac{-g_m R_D}{1 + g_m R_S + \frac{R_D + R_S}{r_d}}}$$

$$G = \left(\frac{v_o}{v_i}\right)^2 = \left(\frac{g_m R_D}{1 + g_m R_S + \frac{R_D + R_S}{r_d}}\right)^2$$

for $r_d >> R_S, R_D$

$g_m R_S >> 1$

$$\frac{v_o}{v_i} = \frac{-g_m R_D}{1 + g_m R_S + \frac{R_D + R_S}{r_d}} = -\frac{R_D}{R_S} \rightarrow \boxed{G = \left(\frac{R_D}{R_S}\right)^2}$$

($1 + g_m R_S \rightarrow g_m R_S$, $\frac{R_D + R_S}{r_d} \rightarrow 0$)